QUANTUM MAGELLAN EFFECT

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Abstract

In this work we consider remarkable experiment of the quantum dynamical interaction between a photon and fixed beam splitter with additional two optical fibers. Given fibers, having "circular", almost completely closed loop forms, admit that both superposition terms, corresponding to reflecting and passing photon, interact unlimitedly periodically with splitter. For increasing number of given interactions final state of the photon tends to superposition of reflecting and passing photon with equivalent superposition coefficients quite independently of their initial values. So, many time repeated unitary quantum dynamical evolution implies an unexpected degeneration. Feynman ingeniously observed that a time of the degeneration of the ideas will come, known to any great geographer-explorer (e.g. Magellan that first circumnavigate Earth), when he thinks about the army of the tourists that will come after him. For this reason mentioned dynamical degeneration will be called quantum Magellan effect. Also, we consider quantum Magellan effect with measurements realized on the photon by movable beam splitter. For increasing number of given measurements photon finally, but slower that by effect without any measurement, tends to mixed state of reflecting and passing photon with equivalent statistical weights quite independently of the initial statistical weights. So, there is again an unexpected quantum dynamical degeneration done slower by frequent measurements. All this is discussed from different aspects including principles of quantum computing.

1 Introduction

In this work we shall consider simple experiment of the quantum dynamical interaction between single photon and fixed beam splitter (e.g. half-silvered mirror, representing a more realistic variant of the well-known, basic experiment of the single photon interference on a diaphragm with two slits [1]-[4]) with additional two optic fibers. Given fibers, having "circular", almost completely (except two points, one for any fiber, in which given fibers touch beam splitter) closed loop forms, admit that both superposition terms, corresponding to reflecting (from beam splitter) and passing

(through beam splitter) photon, interact unlimitedly periodically with beam splitter. When number of given interactions tends toward infinity final state of the photon tends to superposition of reflecting and passing photon with equivalent superposition coefficients quite independently of the initial superposition coefficients. In other words, many time repeated unitary quantum dynamical evolution implies an unexpected degeneration. (By famous quantum Zeno effect [5] unexpected phenomena appears as the result of many times repeated decay measurement. But here, since beam splitter is fixed all appears without any measurement of the photon characteristics, passing or reflecting [3], [4].) Feynman ingeniously observed: "There will be a degeneration of ideas, just like the degeneration that great explorers feel is occurring when tourists begin moving in on a territory."[4]. For this reason mentioned dynamical degeneration will be called quantum Magellan effect, since Magellan, as it is well-known, has been the great explorer that first circumnavigate Earth. Also, we consider quantum Magellan effect with measurements realized on the photon by movable beam splitter [3], [4]. For increasing number of given measurements photon finally, but slower that by effect without any measurement, tends to mixed state of reflecting and passing photon with equivalent statistical weights quite independently of the initial statistical weights. So, there is again an unexpected quantum dynamical degeneration done slower by frequent measurements. All this is discussed from different aspects including principles of quantum computing.

2 Quantum Magellan effect

Consider a photon in state $|\psi_0>$ that, roughly speaking, propagates on the "left" side of a fixed, thin beam splitter (e.g. half-silvered mirror or similar) toward given beam splitter. Since beam splitter is fixed any momentum exchange between photon and splitter is forbidden. For this reason (see for example [3]) interaction between photon and splitter represents pure, unitary quantum dynamical evolution. (Vice versa, for a movable beam splitter, interaction between photon and splitter represents a measurement that can say us is photon reflected or passing trough splitter [3].). Also, this beam splitter is symmetric which means that reflection and transparence coefficients at one side are equivalent to corresponding coefficients at the other side of the splitter. After very short, or, formally speaking, instantaneous, interaction with given beam splitter in, roughly speaking, a point called A_L on the "left" side of beam splitter, initial photon state evolves in the following superposition

$$|\psi_1>= a_1|1> +b_1|0> . \quad (1)$$

Here $|1>$ denotes state of the reflected photon at the "left" side of the splitter, $|0>$ - state of the passing photon that appears, roughly speaking, at the "right" side of the splitter, while a_1 and b_1 represents real superposition coefficients of reflection and transparency that satisfy normalization condition $a_1^2 + b_1^2 = 1$.

Suppose, further, that reflecting photon, or, precisely, photon in state $|1>$ quantum dynamically interacts with optic fiber F_L. Given fiber is placed on the "left" side of beam splitter and touch beam splitter in practically one point A_L. Also, given fiber holds an especial geometry, so that, roughly speaking, it leads reflecting photon along an almost closed (except A_L point) loop trajectory (within this cable) again in the initial state $|\psi_0>$ and further to new interaction with the beam splitter.

Suppose too that passing photon, i.e. photon in state $|0>$, roughly speaking, leaves beam splitter at its "right" side in A_R point corresponding to point A_L. Suppose that given photon quantum dynamically interacts with other optic fiber F_R. Given fiber is placed on the "right"

side of beam splitter and touch beam splitter in practically one point A_R. Also, given fiber holds an especial geometry, so that, roughly speaking, it leads reflected photon along an almost closed (except A_R point) loop trajectory toward A_R point at beam splitter. Then short, or, formally instantaneous, quantum dynamical interaction between given photon and beam splitter appears. Moreover, suppose that geometry of the experimental device (consisting of beam splitter and both fibers) is such that after dynamical interaction with beam splitter previously reflected photon turns out in the superposition with two terms. First one represents the photon, passing splitter from A_R point to A_L point and appearing at the "left" side of the splitter, further propagating along F_L. Second one represents new reflecting photon that reflects in A_R point and that further propagates along F_R. It implies a new interaction with beam splitter.

Picturesquely speaking given experimental scheme is similar to infinity symbol ∞. Here left part of symbol corresponds to F_L, right part of symbol to F_R, and, central point of the symbol to beam splitter (A_L and A_R points). For geography sympathizers both, scheme and infinity symbol, are similar to sketch of Pacific (left) and Atlantic (right) connected with Strait of Magellan.

Suppose, finally, that propagation time of the photon in "left" and "right" fiber is equal and that it equals a finite value T. It means, roughly speaking, that quantum dynamical interaction between photon, beam splitter and fiber is periodical with period T.

It is very important to be repeated and pointed out that, since beam splitter is strongly fixed, here is no any detection, i.e. measurement of the photon character (reflecting or passing).

Thus, according to well-known rules of the quantum mechanics [1], [2], immediately after second interaction with beam splitter, or after time $2T$, photon is in the following superposition

$$|\psi_2> = \frac{1}{(1+4a_1^2b_1^2)^{\frac{1}{2}}}[a_1(a_1|1> + b_1|0>) + b_1(a_1|0> + b_1|1>)] = a_2|1> + b_2|0> \tag{2}$$

where

$$a_2 = \frac{1}{(1+4a_1^2b_1^2)^{\frac{1}{2}}} \tag{3}$$

and

$$b_2 = \frac{2a_1b_1}{(1+4a_1^2b_1^2)^{\frac{1}{2}}} \tag{4}$$

represent corresponding superposition coefficient. It implies that probability of the appearance of the photon in F_L immediately after second interaction with beam splitter equals

$$w_{L2} = a_2^2 = \frac{1}{1+4a_1^2b_1^2} \tag{5}$$

while probability of the appearance of the photon in F_R immediately after second interaction with beam splitter equals

$$w_{R2} = b_2^2 = \frac{4a_1^2b_1^2}{1+4a_1^2b_1^2}. \tag{6}$$

By simple induction it follows that immediately after n-th interaction with beam splitter, or after time nT, photon is in the following superposition

$$|\psi_n> = a_n|1> + b_n|0> \qquad \text{for} \quad n = 2, 3, ... \tag{7}$$

where

$$a_n = \frac{1}{(1+4a_{n-1}^2b_{n-1}^2)^{\frac{1}{2}}} \qquad \text{for} \quad n = 2, 3, ... \tag{8}$$

and

$$b_n = \frac{2a_{n-1}b_{n-1}}{(1+4a_{n-1}^2 b_{n-1}^2)^{\frac{1}{2}}} \qquad \text{for} \quad n = 2, 3, ... \tag{9}$$

represent corresponding superposition coefficient. It implies that probability of the appearance of the photon in F_L immediately after second interaction with beam splitter equals

$$w_{Ln} = a_n^2 = \frac{1}{1+4a_{n-1}^2 b_{n-1}^2} \qquad \text{for} \quad n = 2, 3, ... \tag{10}$$

while probability of the appearance of the photon in F_R immediately after second interaction with beam splitter equals

$$w_{Rn} = b_n^2 = \frac{4a_{n-1}^2 b_{n-1}^2}{1+4a_{n-1}^2 b_{n-1}^2} \qquad \text{for} \quad n = 2, 3, ... \tag{11}$$

Suppose $a_{n-1} = b_{n-1} = 0.5^{\frac{1}{2}} \simeq 0.707$ and $w_{Ln-1} = w_{Rn-1} = 0.500$ for some $n - 1 \geq 1$. Then, according to (8)-(11) it follows $a_n = b_n = 0.5^{\frac{1}{2}} \simeq 0.707$ and $w_{Ln} = w_{Rn} = 0.500$. It means that superposition (7) with equal coefficients is quantum dynamically stable and that it, further, practically does not evolve during time.

But, what happens in other situations when superposition coefficients are initially sufficiently different?

We shall demonstrate answer by use of two concrete example considered numerically.

Suppose $w_{L1} = a_1^2 = 0.9$ and $w_{R1} = b_1^2 = 0.1$, or, correspondingly, $a_1 = 0.949$ and $b_1 = 0.316$. Then, according to (8)-(11) it follows $w_{L2} = 0.735$ and $a_2 = 0.857$, $w_{L3} = 0.562$ and $a_3 = 0.750$, $w_{L4} = 0.504$ and $a_4 = 0.709$, and, $w_{L5} = 0.500$ and $a_5 = 0.707$. It simply points out that already for $n = 5$ state of the photon becomes stable with error smaller than 10^{-4}.

Suppose $w_{L1} = a_1^2 = 0.05$ and $w_{R1} = b_1^2 = 0.95$, or, correspondingly, $a_1 = 0.224$ and $b_1 = 0.975$. Then, according to (8)-(11) it follows $w_{L2} = 0.84$ and $a_2 = 0.917$, $w_{L3} = 0.65$ and $a_3 = 0.806$, $w_{L4} = 0.524$ and $a_4 = 0.724$, and, $w_{L5} = 0.5006$ and $a_5 = 0.7075$. It simply points out that already for $n = 5$ state of the photon becomes stable with error smaller than $5 \cdot 10^{-4}$.

All this implies that in a large domain of the superposition coefficients and corresponding probabilities values initial superposition tends to final stable superposition for practically very small number (proportional or smaller than 5) of the repetitions of interaction. It represents an unexpected result.

Firstly, different values of superposition coefficients correspond to different quantum dynamical interactions or unitary operators that act on the same photon initial state $|\psi_0>$. For this reason, according to standard quantum mechanical formalism, final photon states, corresponding to different dynamical interactions, must be different too. It is exactly satisfied, but when number of the interaction increases any of final photon states tends to the same stable superposition. In this sense we have an approximate, asymptotic dynamical degeneracy. For this reason, according to mentioned Feynman observation, given effect can be called quantum Magellan effect.

Secondly, quantum Magellan effect appears effectively very quickly, i.e. for very small increase of the number of the interactions. For instance, quantum Zeno effect needs much larger number of the decay measurements.

3 Quantum Magellan effect with measurements

Suppose that in the mentioned experimental scheme of the quantum Magellan effect beam splitter is no more fixed, but movable which admits controllable exchange of the momentum by interaction between photon and splitter. For this reason (see for example [3], [4]) interaction between photon and splitter represents a measurement - M that breaks superposition and can say us is photon reflected or passing trough splitter.

Then, as it is not hard to see, according to well-known rules of the quantum mechanics [1], [2], immediately after n-th interaction with beam splitter, or after time nT, photon is in the mixture of reflected and passing state, i.e. $|1>$ and $|0>$ with corresponding probabilities

$$w_{LMn} = a_1^2 w_{LMn-1} + b_1^2 w_{RMn-1} \qquad \text{for} \quad n = 2, 3, ... \tag{12}$$

$$w_{RMn} = 1 - w_{LMn} \qquad \text{for} \quad n = 2, 3, ... \tag{13}$$

where

$$w_{LM1} = a_1^2 \tag{14}$$

$$w_{RM1} = b_1^2. \tag{15}$$

Suppose $w_{LMn-1} = w_{RMn-1} = 0.5$ for some $n - 1 \geq 1$. Then, according to (12), (13), it follows $w_{LMn} = w_{RMn} = 0.5$. It means that given mixture is quantum dynamically stable and that it, further, practically does not evolve during time.

In other situations, when w_{LM1} is sufficiently different from w_{RM1}, simple calculations according to (12), (13), point out that when n increases w_{LMn} and w_{RMn} tend toward 0.5. For example for $w_{LM1} = a_1^2 = 0.9$ and $w_{RM1} = b_1^2 = 0.1$ it follows $w_{LM2} = 0.820$, $w_{LM3} = 0.7552$, $w_{LM4} = 0.703$, and, $w_{LM5} = 0.661$. Obviously, given probabilities tend to 0.5 slower than in the above discussed situation of quantum Magellan effect without measurement with the same initial probabilities. This inhibition of the quantum dynamical effects, i.e. tendency of the probabilities toward 0.5 by frequent measurement, represents, of course, some kind of partial quantum Zeno effect [5].

But, in fact, all this can be interpreted in other way.

By movable beam splitter, photon behaves similarly to a classical particle (without wave, i.e. interference characteristics) [3], [4]. Then, roughly speaking, whole mentioned experimental scheme can be considered as a classical machine for realization of the classical algorithm for final equivalence (equilibrium) of initially different inputs (statistical weights) in a relatively large number of the steps (number of the measurements of the photon by interaction with movable beam splitter).

By fixed beam splitter photon behaves as a quantum wave (with interference characteristics). Then whole mentioned experimental scheme can be considered as a quantum machine for realization of the quantum algorithm for final equivalence (equilibrium) of the initially different inputs (superposition coefficients) in not so large number of the steps (number of the quantum dynamical interactions between photon and beam splitter).

All this simply demonstrates faster work of given quantum in respect to classical machine for equivalence algorithm realization. It can be interesting for quantum computing.

4 Some variations of quantum Magellan effect

Suppose now that experimental scheme for quantum Magellan effect without measurements is changed in the following way. Namely, suppose that F_R input and beam splitter stand connected

in A_R point. Meanwhile suppose that now F_R output is not connected with F_R input and beam splitter in A_R point, but that F_R output is connected with some other point of F_R and disconnected with beam splitter. It can be simply called half-disconnection or half-connection between F_R and beam splitter. In this way, for passing photon, F_R becomes completely closed loop which completely captures this photon. Then only reflecting photon can further periodically quantum dynamically interact with beam splitter.

It is not hard to see that now, after n-th such interaction, photon is described by the following superposition

$$|\psi_n>= a_n|1> +b_n|0> \qquad \text{for} \quad n = 2, 3, ... \tag{16}$$

where

$$a_n = \frac{a_{n-1}^2}{(a_{n-1}^4 + b_{n-1}^2(1 + a_{n-1})^2)^{\frac{1}{2}}} \qquad \text{for} \quad n = 2, 3, ... \tag{17}$$

and

$$b_n = \frac{b_{n-1}(1 + a_{n-1})}{(a_{n-1}^4 + b_{n-1}^2(1 + a_{n-1})^2)^{\frac{1}{2}}} \qquad \text{for} \quad n = 2, 3, ... \tag{18}$$

represent corresponding superposition coefficient. It implies that probability of the appearance of the photon in F_L immediately after second interaction with beam splitter equals

$$w_{Ln} = a_n^2 = \frac{a_{n-1}^4}{a_{n-1}^4 + b_{n-1}^2(1 + a_{n-1})^2} \qquad \text{for} \quad n = 2, 3, ... \tag{19}$$

while probability of the appearance of the photon in F_R immediately after second interaction with beam splitter equals

$$w_{Rn} = b_n^2 = \frac{b_{n-1}^2(1 + a_{n-1})^2}{a_{n-1}^4 + b_{n-1}^2(1 + a_{n-1})^2} \qquad \text{for} \quad n = 2, 3, ... \tag{20}$$

Simple numerical calculations point out that when number of the interactions n increases, superposition coefficient and probability a_n and w_{Ln} tend toward zero, while superposition coefficient and probability b_n and w_{Rn} tend toward one, practically independently of the initial values of the superposition coefficients. Thus, we obtain again a quantum dynamical degeneration or quantum Maggelan effect.

Suppose, further, that in given experimental scheme beam splitter is movable so that interaction between photon and beam splitter can be considered as the photon measurement - M. Then, as it is not hard to see, after n-th interaction, i.e. measurement, photon is described by the mixture of the passing and reflecting photons with statistical weights

$$w_{LMn} = a_1^2 w_{LMn-1} \qquad \text{for} \quad n = 2, 3, ... \tag{21}$$

and

$$w_{RMn} = b_1^2 w_{LMn-1} + w_{RMn-1} \qquad \text{for} \quad n = 2, 3, ... \tag{22}$$

Simple numerical calculations point out that when number of the interactions, i.e. measurements n increases, statistical weight of reflecting photon w_{LMn} decreases and tends toward zero, while statistical weight of the passing photon w_{RMn} increases and tends toward one, independently of the initial statistical weights w_{LM1} and w_{RM1}. In this way we obtain degeneracy characteristic for quantum Maggelan effect with measurements.

Also, simple numerical calculations point out that quantum Maggelan effect without measurement appears faster than quantum Maggelan effect without measurement.

Finally, it is not hard to see that corresponding situation we obtain by such experimental scheme of quantum Maggelan effect when half-disconnection between F_L and beam splitter is done.

In this way, for reflecting photon, F_L becomes completely closed loop which completely captures this photon. Then only passing can further periodically quantum dynamically interact with beam splitter.

In this case, for fixed beam splitter, by increase of the quantum dynamical interaction between photon and beam splitter, we obtain final pure state corresponding to reflecting photon independent of initial superposition coefficient a_1 and b_1. It represents dynamical degeneration characteristic for quantum Maggelan effect without measurement.

For movable beam splitter, by increase of the number of interactions between photon and beam splitter, or measurements of the photon by beam splitter, we obtain final statistical mixture of reflecting photons independently of initial statistical weights w_{LM1} and w_{RM1}. In this way we obtain degeneracy characteristic for quantum Maggelan effect with measurements.

Also, simple numerical calculations point out that here again quantum Maggelan effect without measurement appears faster than quantum Maggelan effect without measurement.

5 Discussion and conclusion

Quantum Magellan effect without measurement in all three mentioned cases (both fibers connected with beam splitter, "left" fiber connected and "right" half-connected with beam splitter, and, "left" fiber half-connected and "right" fiber connected with beam splitter) represents an interesting quantum phenomenon. Obviously, it is practically completely caused by "topology" of the experimental scheme, i.e. "topology" of the quantum dynamics. It means that given effect depends of (almost complete) closed loop photon trajectory in both optical fibers and connection and half-connection of given trajectories. Also, given effect depends of the quantum normalization condition, i.e. condition of the "topological" continuity of the norm of the quantum states. On the other hand this effect is practically independent of the "geometry" of the experimental scheme, i.e. "geometry" of the quantum dynamics. It means that given effect is practically independent of the initial values of the superposition coefficients (in cases without measurements) or statistical weights (in cases with measurements). All this implies a possible analogy. Namely, it seems that quantum Maggelan effect by both optical fibers with almost completely closed loop form, i.e. by complete connection of both optical fiber with beam splitter, is, at least in some degree similar to remarkable Möbius strip (tape).

Further, as it has been demonstrated previously, in quantum Magellan effect without measurements by both connected fibers arbitrary initial superposition turns out finally practically certainly in the stable superposition with equivalent coefficients. Also, as it is demonstrated, in quantum Magellan effect without measurements by one half-connected fiber that arbitrary initial superposition turns out finally practically certainly in the state corresponding to half-connected fiber. All this has very important implications. Namely simple, fast (with duration many time smaller than T), external manipulation with character of the fibers connection (change of a half-connected in connected and vice versa) can dynamically and deterministically change state of the photon in somewhat unexpected way. Initial stable superposition (with equivalent superposition

coefficients), by half-disconnection of one optical fiber, turns out dynamically and practically certainly in final state in half-disconnected fiber. Vice versa, initial state in half-disconnected fiber, by complete connection of given fiber, turns out dynamically and practically certainly in final stable superposition (with equivalent superposition coefficients) In this way, formally speaking, we obtain, within standard quantum mechanical formalism, a procedure for getting of the eigen states from superposition most efficient than corresponding measurement. It can be very interesting for application in many domains of the quantum physics, e.g. for quantum computing.

In conclusion we can shortly repeat and point out the following. In this work we considered remarkable experiment of the quantum dynamical interaction between a photon and fixed beam splitter with additional two optical fibers. Given fibers, having "circular", almost completely closed loop forms, admit that both superposition terms, corresponding to reflecting and passing photon, interact unlimitedly periodically with splitter. For increasing number of given interactions final state of the photon tends to superposition of reflecting and passing photon with equivalent superposition coefficients quite independently of their initial values. So, many time repeated unitary quantum dynamical evolution implies an unexpected degeneration. For this reason, and according to an ingenious Feynman observation, mentioned dynamical degeneration is called quantum Magellan effect. All this, including cases with movable beam splitter, is discussed from different aspects including principles of quantum computing.

6 References

[1] P. A. M. Dirac, *Principles of Quantum Mechanics* (Clarendon Press, Oxford, 1958)

[2] R. P. Feynman, R. B. Leighton, M. Sands, *The Feynman Lectures on Physics, Vol. 3* (Addison-Wesley Inc., Reading, Mass. 1963)

[3] N. Bohr, *Atomic Physics and Human Knowledge* (John Wiley, New York , 1958)

[4] R. Feynman, *The Character of Physical Law* (Cox Wyman LTD, London, 1965)

[5] B. Misra, C. J. G. Sudarshan, J. Math. Phys. **18** (1977) 756